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Comparison of Approaches to Quantum Gravitational Corrections on the Reissner-Nordström-de Sitter Black Hole Tunneling Radiation

M. Dehghani · A. Farmany

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Abstract Although the tunneling approach is fully established for black hole radiation, much work has been done to support the extension of this approach to more general settings. In this letter the Parikh-Kraus-Wilczeck tunneling proposal of black hole tunneling radiation is considered. The Reissner-Nordström-de Sitter black hole thermodynamics is studied according to the generalized uncertainty principle and the modified dispersion relation analysis. It is shown that entropy, temperature and the original Parikh-Kraus-Wilczek calculations of the black hole tunneling probability receive new corrections. The results are compared and it is shown that these two alternative approaches lead to the same results if one uses the suitable expansion coefficients.

Keywords Black hole radiation · Black hole thermodynamics · Tunneling radiation · Generalized uncertainty principle · Modified dispersion relation · Reissner-Nordström-de Sitter black hole

1 Introduction

In 1974, Hawking proved that a black hole radiates particles and the radiation spectrum is purely thermal, which plays a key role in the cognation and research on black holes [1, 2]. Recently, Wilczeck et al. have provided an alternative method to drive the Hawking radiation of the black holes. Their method was based on the semi-classical tunneling [3–8], and received much attention [9–16]. In this method, derivation of the Hawking temperature is related to the imaginary part of action for classically forbidden process of s-wave emission across the horizon. The imaginary part of the action for emitted particles is calculated using different methods. In the null-geodesic method, the contribution to the imaginary part of the

M. Dehghani (✉)
Department of Physics, Ilam University, Ilam, Iran
e-mail: dehghan22@gmail.com

A. Farmany
Department of Chemistry, Islamic Azad University of Ilam, Ilam, Iran

action comes from the integration of radial momentum of the emitted particles. Other approaches satisfy the relativistic Hamilton-Jacobi equation of the emitted particles to obtain the imaginary part of action [3–12, 17]. As a comparison between Hawking original calculation and tunneling method, it is easy to see that the Hawking method is a direct method but its complication to generalization to all other space times is failed while the tunneling approaches have been successfully applied to a wide range of both the black hole horizons and cosmological horizons.

Recently generalized uncertainty principle (GUP) has been the subject of many interesting works and a lot of papers have appeared in which that usual uncertainty is modified at the framework of microphysics [18–25]. To study the quantum gravitational effects on the tunneling mechanism it is interesting to relate the tunneling analysis with a minimal length quantum gravity scale.

The possibility that the usual relation between the energy and momentum in special relativity may be modified at Planck scale, conventionally named as modified dispersion relations (MDR's). The modification of energy-momentum relations and it's implications have been investigated extensively by many authors [26–29].

Study of thermodynamics properties of de Sitter and asymptotically de Sitter space-times may be interesting because based on the recent astronomical observation of supernova, the cosmological constant may be positive [30, 31]. This indicates that our universe might approach a de Sitter phase in the far future.

In this letter, we first extend the tunneling black hole radiation probability based on the GUP analysis. Using the GUP, we calculate the quantum gravitational effects on the entropy, temperature and tunneling radiation through the horizon of Reissner-Nordström-de Sitter black hole, up to the fourth order in the Planck length. Then by using the MDR, we calculate the corrected entropy, temperature and tunneling probability through horizon of the black hole. Finally, we compare the results of these two alternative approaches and show that a suitable choice of the expansion coefficients leads to the same results for the black hole thermodynamics and tunneling radiation.

2 Hawking Radiation as Tunneling

According to tunneling picture, the radiation arises by a process similar to electron-positron pair production in a constant electric field. The idea is that the energy of a particle changes sign as it crosses the horizon, so that the pair created just inside or outside the horizon can materialize with zero total energy, after each one of the pairs has tunneled to the opposite sides [3–8]. This suggests that it should be possible to describe the black hole emission process in a semiclassical fashion as quantum tunneling. In the WKB approximation, the tunneling probability is a function of the imaginary part of the action [32–35]

$$\Gamma \sim e^{-2\mathcal{I}_m(I)}, \tag{2.1}$$

where $\mathcal{I}_m$ is the imaginary part and I is the classical action of the trajectory.

Here, we consider the Reissner-Nordström-de Sitter black hole with standard line element,

$$ds^2 = -\left(1 - \frac{2GM}{r} + \frac{Q^2}{r^2} - \frac{\Lambda}{3}r^2\right)dt^2 + \left(1 - \frac{2GM}{r} + \frac{Q^2}{r^2} - \frac{\Lambda}{3}r^2\right)^{-1} dr^2 + r^2 d\Omega^2, \tag{2.2}$$

where $\Lambda = 3H^2$ is the positive cosmological constant. There is an event horizon situated at r_+, which comes from $1 - \frac{2GM}{r_+} + \frac{Q^2}{r_+^2} - \frac{\Lambda}{3}r_+^2 = 0$.

In the following, we restrict ourselves to uncharged particles radiation from the horizon of the Reissner-Nordström-de Sitter black hole. Note that if one considers the charged radiated particles, the trajectories are also subject to electromagnetic forces. To describe the tunneling phenomena, we need the coordinates which, unlike the coordinates given in (2.2), are not singular on the horizon. For this purpose, we use the following Painlevé-like coordinate transformations [36] in (2.2),

$$dt_r = dt - \frac{\sqrt{\frac{2GM}{r} - \frac{Q^2}{r^2} + \frac{\Lambda}{3}r^2}}{1 - \frac{2GM}{r} + \frac{Q^2}{r^2} - \frac{\Lambda}{3}r^2} dr, \tag{2.3}$$

where t_r is the Reissner time coordinate. This coordinate transformation leads to the non-singular nature of the space-time with the following line element

$$ds^2 = -\left(1 - \frac{2GM}{r} + \frac{Q^2}{r^2} - \frac{\Lambda}{3}r^2\right) dt^2 + 2\sqrt{\frac{2GM}{r} - \frac{Q^2}{r^2} + \frac{\Lambda}{3}r^2} dr dt + dr^2 + r^2 d\Omega^2. \tag{2.4}$$

In the null geodesic method, the imaginary part of the action for an s-wave outgoing positive energy particle which crosses the horizon outward from r_{in} to r_{out} comes from the radial part of the momentum

$$\begin{aligned}\mathcal{I}_m(I) &= \mathcal{I}_m \int_{r_{\text{in}}}^{r_{\text{out}}} p_r dr = \mathcal{I}_m \int_{r_{\text{in}}}^{r_{\text{out}}} \int_0^{p_r} dp_r dr \\ &= \mathcal{I}_m \int_M^{M-\omega} \int_{r_{\text{in}}}^{r_{\text{out}}} \frac{dr}{\dot{r}} dH,\end{aligned} \tag{2.5}$$

where the Hamilton's equation $\dot{r} = dH/dp_r$ is used. The radial null geodesic for an outgoing massless particle is given by

$$\dot{r} = 1 - \sqrt{\frac{2GM}{r} - \frac{Q^2}{r^2} + \frac{\Lambda}{3}r^2}. \tag{2.6}$$

If we fix the total mass and let the black hole mass to fluctuate, a shell of energy ω travels on the geodesic given by line element (2.4) with $M \to M - \omega$. Using (2.6) in (2.5) and switch the order of integration we have

$$\mathcal{I}_m(I) = -\int_0^{\omega} \int_{r_{\text{in}}}^{r_{\text{out}}} \frac{dr}{1 - \sqrt{\frac{2G(M-\omega')}{r} - \frac{Q^2}{r^2} + \frac{\Lambda}{3}r^2}} d\omega' \tag{2.7}$$

which can be calculated by deforming the contour according to Faynman's $\omega' \to \omega' + i\epsilon$ prescription. This results the black hole radiation probability as

$$\Gamma \sim e^{-2\mathcal{I}_m(I)} = e^{\Delta S_{B-H}}, \tag{2.8}$$

where $\Delta S_{B-H} = S_{B-H}(M-\omega) - S_{B-H}(M)$ is the difference between the initial and the final values of the Beckenstein-Hawking entropy of the Reissner-Nordström-de Sitter black hole.

The Reissner-Nordström-de Sitter black hole horizon has an associated entropy and temperature as

$$S = \frac{A}{4G}, \tag{2.9}$$

$$T = \frac{1}{2\pi r_+}\left(\frac{GM}{r_+} - \frac{Q^2}{r_+^2} - \frac{\Lambda}{3}r_+^2\right). \tag{2.10}$$

The area and mass are related to the black hole horizon through

$$A = 4\pi r_+^2, \qquad 1 - \frac{2GM}{r_+} + \frac{Q^2}{r_+^2} - \frac{\Lambda}{3}r_+^2 = 0, \tag{2.11}$$

from which we obtain

$$dA = 8\pi r_+ dr_+, \qquad dM = \left(\frac{GM}{r_+} - \frac{Q^2}{r_+^2} - \frac{\Lambda}{3}r_+^2\right)\frac{dr_+}{G}. \tag{2.12}$$

3 GUP and Black Hole Thermodynamics

To study the quantum gravity effects on the tunneling probability, we employ the GUP. Recently, the GUP has been the subject of many interesting works. It is shown that usual uncertainty principle receives a modification at the microphysics regime [37–44],

$$\Delta x \geq \frac{\hbar}{\Delta p} + \alpha L_P^2 \frac{\Delta p}{\hbar}, \tag{3.1}$$

where L_p is the Planck length. The term $\alpha L_P^2 \frac{\Delta p}{\hbar}$ in (3.1) shows the gravitational effects on the usual uncertainty principle. Inverting (3.1) we obtain

$$\frac{\Delta x}{2\alpha L_P^2}\left(1 - \sqrt{1 - \frac{4\alpha L_P^2}{(\Delta x)^2}}\right) \leq \frac{\Delta p}{\hbar} \leq \frac{\Delta x}{2\alpha L_P^2}\left(1 + \sqrt{1 - \frac{4\alpha L_P^2}{(\Delta x)^2}}\right). \tag{3.2}$$

From (3.2), one can write

$$\left(\frac{\Delta p}{\hbar}\right)_{\min} = \frac{\Delta x}{2\alpha L_P^2}\left(1 - \sqrt{1 - \frac{4\alpha L_P^2}{(\Delta x)^2}}\right). \tag{3.3}$$

Expanding (3.3) around $L_P = 0$ gives,

$$\Delta p_{\min} = \frac{\hbar}{\Delta x}\left[1 + \frac{\alpha L_P^2}{(\Delta x)^2} + 2\frac{\alpha^2 L_P^4}{(\Delta x)^4} + 5\frac{\alpha^3 L_P^6}{(\Delta x)^6} + \cdots\right]. \tag{3.4}$$

Increasing the black hole area by absorbing a particle of energy $cdp \approx dM$ may be expressed as

$$dA = 8\pi r_+ dr_+ = \frac{8\pi G r_+ dp}{\frac{GM}{r_+} - \frac{Q^2}{r_+^2} - \frac{\Lambda}{3}r_+^2}. \tag{3.5}$$

Using the quantum gravitational effects in (3.5) through (3.4), we obtain

$$dA_{(\mathrm{GUP})} = \left[1 + \frac{\alpha L_P^2}{(\Delta x)^2} + 2\frac{\alpha^2 L_P^4}{(\Delta x)^4} + 5\frac{\alpha^3 L_P^6}{(\Delta x)^6} + \cdots\right] dA. \tag{3.6}$$

A particle that is absorbed by the black hole horizon area has a Compton length as

$$\Delta x = 2r_+ = 2\sqrt{\frac{A}{4\pi}}. \tag{3.7}$$

Substituting (3.7) and (2.6) leads to

$$A_{(\mathrm{GUP})} = A + \pi\alpha L_P^2 \ln A - \frac{2\pi^2\alpha^2 L_P^4}{A} + \cdots + \mathrm{Const}. \tag{3.8}$$

Combining (3.8) with the Bekenstein-Hawking entropy/area relation, we obtain the generalized Bekenstein-Hawking entropy of black hole based on the GUP as,

$$S_{(\mathrm{GUP})} = \frac{A}{4L_P^2} + \frac{\pi\alpha}{4}\ln\left(\frac{A}{4L_P^2}\right) - \frac{\pi^2\alpha^2}{8}\left(\frac{A}{4L_P^2}\right)^{-1} + \cdots + \mathrm{Const}, \tag{3.9}$$

$$= S + \frac{\pi\alpha}{4}\ln S - \frac{\pi^2\alpha^2}{8}(S)^{-1} + \cdots + \mathrm{Const}. \tag{3.10}$$

Using (3.9) and $dM = TdS$, one can obtain the generalized Hawking temperature of the black hole,

$$T_{(\mathrm{GUP})} = T\left(1 - \frac{\pi\alpha}{A}L_P^2 - \frac{\pi^2\alpha^2}{A^2}L_P^4\right), \tag{3.11}$$

up to the fourth order of the Planck length. To obtain the tunneling probability of particle emission of black hole, (3.10) must be substituted in (2.8)

$$\Gamma_{(\mathrm{GUP})} \sim \exp\left(S + \frac{\pi\alpha}{4}\ln S - \frac{\pi\alpha^2}{8S} + \cdots\right) = \Gamma\exp\left(\frac{\pi\alpha}{4}\ln S - \frac{\pi\alpha^2}{8S} + \cdots\right). \tag{3.12}$$

Equation (3.12) is the corrected black hole's radiation tunneling probability based on the GUP.

4 MDR and Black Hole Thermodynamics

In the study on loop quantum gravity and of models based on noncommutative geometry, there has been a strong interest in some candidate modifications of the energy-momentum dispersion relation. In this section, we consider the effects of MDR on the de Sitter-Schwarzschild black hole thermodynamics and tunneling radiation probability.

It is interesting that the usual relation between energy and momentum that characterizes the special theory of relativity, $p^2 = E^2 - m^2$, may be modified in the Planck scale regime. Anomalies in ultra high cosmic ray photons, and possibly TeV photons, may be explained by modification of the dispersion relation as [45–48]

$$\vec{p}.\vec{p} \equiv p^2 = f(E, m; L_P) = E^2 - \mu^2 + \alpha_1 L_P^2 E^4 + \alpha_2 L_P^4 E^6 + \mathcal{O}\left(L_P^6 E^8\right), \tag{4.1}$$

where f is the function that gives the exact dispersion relation, and on the right-hand side we have assumed the applicability of a Taylor-series expansion for $E \ll \frac{1}{L_P}$. The coefficients

α_i can take different values in different quantum-gravity proposals. Note that m is the rest energy of the particle and the mass parameter μ on the right-hand side is directly related to the rest energy, but $\mu \neq m$, if the α_i do not all vanish. Now differentiation of (4.1) and taking the inverse of the result gives

$$dE = dp\left[1 - \frac{3\alpha_1}{2}L_P^2E^2 - \left(\frac{5\alpha_2}{2} - \frac{23\alpha_1^2}{8}\right)L_P^4E^4\right]. \tag{4.2}$$

Within quantum field theory, the relation between particle localization and its energy is given by $E \geq \frac{1}{\Delta x}$, where Δx is particle position uncertainty. Now, it is obvious that within MDR, this relation should be modified. In a simple analysis based on the familiar derivation of the relation $E \geq \frac{1}{\Delta x}$, one can obtain the corresponding generalized relation as follows:

$$E\Delta x \geq 1 - \frac{3\alpha_1}{2}\frac{L_P^2}{(\Delta x)^2} - \left(\frac{5\alpha_2}{2} - \frac{23\alpha_1^2}{8}\right)\frac{L_P^4}{(\Delta x)^4}. \tag{4.3}$$

As shown by Amelino-Camelia et al. [49], the MDR may open the possibility of studying the black hole thermodynamics. It is interesting that in addition to GUP, MDR may be used to study Planck scale corrections to the black hole thermodynamics and tunneling radiation.

Consider a quantum particle with energy E and size l is absorbed into a black hole and $l \sim \Delta x$, the minimum increase of area of black hole will be

$$\Delta A = 4L_P^2 E\Delta x \ln 2, \tag{4.4}$$

one can obtain

$$\frac{dS}{dA} \approx \frac{\Delta S_{\min}}{\Delta A_{\min}} \simeq \frac{1}{4L_P^2 E\Delta x}. \tag{4.5}$$

Combining (4.3), (4.5) and (3.7), we obtain

$$S_{(\mathrm{MDR})} \simeq \frac{A}{4L_P^2} + \frac{3\pi\alpha_1}{8}\ln\left(\frac{A}{4L_P^2}\right) - \left(\alpha_2 - \frac{\alpha_1^2}{4}\right)\frac{5\pi^2}{32}\left(\frac{A}{4L_P^2}\right)^{-1} + \mathrm{Const}, \tag{4.6}$$

$$= S + \frac{3\pi\alpha_1}{8}\ln S - \left(\alpha_2 - \frac{\alpha_1^2}{4}\right)\frac{5\pi^2}{32}(S)^{-1} + \mathrm{Const}. \tag{4.7}$$

The first term in (4.6) and (4.7) is the conventional Bekenestein-Hawking entropy and the other terms are corrections based on the MDR.

Using (4.6), one can obtain the modified Hawking temperature of the black hole based on the MDR as

$$T_{(\mathrm{MDR})} = T\left[1 - \frac{3\pi\alpha_1}{2A}L_P^2 - \frac{(5\alpha_2 - \frac{23}{4}\alpha_1^2)\pi^2}{2A^2}L_P^4\right], \tag{4.8}$$

up to the fourth order of the Planck length. Now using (4.8) in (2.8), we have

$$\Gamma_{(\mathrm{MDR})} \sim \exp\left[S + \frac{3\pi\alpha_1}{8}\ln S - \left(\alpha_2 - \frac{\alpha_1^2}{4}\right)\frac{5\pi^2}{32}(S)^{-1} + \cdots\right]$$

$$= \Gamma\exp\left[\frac{3\pi\alpha_1}{8}\ln S - \left(\alpha_2 - \frac{\alpha_1^2}{4}\right)\frac{5\pi^2}{32}(S)^{-1} + \cdots\right]. \tag{4.9}$$

Equation (4.9) is the corrected black hole radiation tunneling probability based on the MDR.

If we require the results of the two given approaches to be consistent, the coefficients must satisfy the following relations

$$\alpha_1 = \frac{2}{3}\alpha, \quad \text{and} \quad \alpha_2 = \frac{41}{45}\alpha^2. \tag{4.10}$$

Using these relations in (3.9)–(3.12) and (4.6)–(4.9), one can show that

$$S_{(\mathrm{MDR})} = S_{(\mathrm{GUP})}, \quad \text{and} \quad T_{(\mathrm{MDR})} = T_{(\mathrm{GUP})}, \quad \text{and} \quad \Gamma_{(\mathrm{MDR})} = \Gamma_{(\mathrm{GUP})}. \tag{4.11}$$

It seems that GUP and MDR approaches lead to the same results for the black hole thermodynamics and tunneling radiation.

5 Conclusion

The Reissner-Nordström-de Sitter black hole thermodynamics and the quantum tunneling radiation probability through the horizon of the black hole are analyzed in the presence of (1) the generalized uncertainty principle and (2) the modified dispersion relation. It is shown that the black hole temperature, entropy and quantum tunneling probability receive corrections. The corrections are calculated up to the fourth order in the Planck length. Through the comparison of the corrected results obtained from these two alternative approaches, it is shown that a suitable choice of the expansion coefficients in the modified dispersion relation leads to the same results in both approaches.

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